Production Systems and Information Engineering
Volume 5 (2009), pp. 95-108.

AN EXTENDED NEWSVENDOR MODEL FOR SOLVING CAPACITY CONSTRAINT PROBLEMS IN A MULTI-ITEM, MULTI-PERIOD ENVIROMENT

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[Received January 2009 and accepted April 2009]

Abstract. In the past few years, the effective management of inventory control problems has become an increasingly critical issue for supplier companies. In this paper, on the basis of the needs a major Hungarian mass production company, we present an extension of an analytical inventory control model considering the condition of global capacity constraint. In a previous paper [6] we elaborated a model regarding the one customer - one supplier relation. Our aim is to determine an optimal holding-production policy of the supplier, which makes a cost-optimum stockpiling policy possible for an arbitrary long production time. We intend to show that, on the basis of our former results, the global capacity constraint satisfying policy can be determined with a new heuristic method.

Keywords: stockpiling policy, extended newsvendor model, global capacity constraint

1. Introduction

In the past 15 years, the business environment of companies in the field of mass production has changed. The demand for mass products has remained high but numerous new requirements have appeared on the market. Changes in the business environment influence engineering and logistic relations between companies and suppliers. The former, simple buying-selling (so-called 'cool') relation has become much 'warmer'. This means that cooperative and collaborative methods and activities have become the main objectives in SCM development. Relations between marketing organizations, end-product manufacturers and supplier companies can be very complicated and diverse in practice. This motivates a wide examination of the available models and further investigation of effective decision supporting and planning methods.

The professional literature includes a wide variety of stockpiling models [6]. Later we will deal with one of the best known stochastic methods, the so-called 'newsvendor model'. The model is certainly among the most important models in the field of operations management. It is applied in a wide variety of stockpiling problems. In this paper we will examine an extended newsvendor model [7] on the basis of former results in a multi-product and capacity constraint case.

The properties of our extended newsvendor model make it possible to solve the inventory control problem of an arbitrary length production horizon can be solved analytically, which opens up new opportunities to model multi-product capacity constraint problems. Capacity constraint problems appear in almost all larger or smaller manufacturing companies. These firms often produce several products meeting customer demands. In case of dynamically and stochastically changing demands, capacity problems often appear. The question is always the same: how much should be produced? Of course the question is very simple, but the solution always belongs to type NP-hard.

The model conditions are identical to the conditions outlined in publications [5,6]. The larger part of the models solves the problem applying the dynamic programming or some kind of searching method (soft-computing). Due to the large searching space, these solutions require extremely long computing times in case of many products and long production time horizon. In our research we investigated capacity constraint problems ranging from one-product one-period to multi-products and multi-periods. This paper aims to present only some of these methods.

1.1 Related studies

Modelling and solving inventory control problems demand for efficiently has been existed since the establishment of the first industrial companies, factories and enterprises. The first successful publications appeared at the beginning of the 1950's. Since then a great number of papers have been published on stockpiling, which proves that the subject is up-to-date. The most important events related to the evolution of inventory control models are fully summarized in the paper by Hans-Joachim Girlich and Attila Chikán [1999]. The main stream research results are concerned with one-product, one-period deterministic models. These models aim to give an optimum policy in an analytical way in accordance with the objective function of the modelled reality. Multi-period deterministic and stochastic models applying multi-products were developed only in later years.

Another way of carrying out stockpiling policies is game theory approaches. Game theory provides effective methods for modelling the 'warming-up' process of the supplier – end manufacturer and customer – vendor relations, which tend to be ever closer nowadays as well as for modelling their cooperation. Next the results of the past nearly 50 years are surveyed, an outstanding example being John von Neumann and Oskar Morgenster's famous book, the "Theory of Games and Economic Behavior" [16], which gave a new direction to the approach of inventory problems.

In the late 1950s, the problem of 'Optimal Inventory Policy' was analyzed by two important economists: Arrow and Marschak [13]. Karlin solved this problem with a dynamic programming method (The Structure of Dynamic Programing Models) [14]. Thirty-six years later, Alistair Milne [15] emphasized that one of the best papers in the area of production decisions and inventory analysis was the study by Arrow, Karlin and Scarf entitled 'Studies in the Mathematical Theory of Inventory and Production' [11]. Among the deterministic models, the Wagner-Within method minimizing the total cost plays an important role. It determines the optimal inventory level with O(n logn) calculation time for an n length finite time horizon.

The paper by Dvoretzky, Kiefer and Wolfowitz [17] examined the (S,s) type policy in the case of a fixed time interval and penalty cost. Nowadays the analysis of inventory-holding problems has become an important part of the management of supply chains. Many excellent publications have appeared concerning this topic [9,10,12], which apply the deterministic demand model [5].

Nowadays, regarding supply chain problems, the most prominent results are linked with the name of G.P. Cachon [2,3]. Stockpiling plays an important role in the management of supply chains. With the rapid evolution of information technology, ERP (Enterprise Resource Planning) and SCM (Supply Chain Management) application systems are gaining significance. Dynamic systems with many products are manageable with operations research models or constraint programming methods. However, solutions based on analytical results and heuristics are decisive in 'what if' type investigations and in the case of quick decisions.

1.2 An Extended Newsvendor Model

The classic newsvendor model cannot be applied properly to solve the tasks of customized mass production. The reason for this is high setup costs that cannot tackle multi-period problems where customer demand can vary stochastically. During our research we developed a new inventory control method, which gives the optimal solution for the problem in an analytic way, and ensures efficient stockpiling for the supplier.

Summarizing the main features of the model the objective function can be formulated as follows:

$$K_{123\ldots n}(q) = c_f + c_v[q - I] + hE[q - D_1]^+ + hE[q - D_1 - D_2]^+ + \ldots +$$
$$+ hE[q - D_1 - D_2 - \ldots - D_n]^+ + pE[D_1 - q]^+ + pE[(D_1 + D_2) - q]^+ + \ldots + \quad (1.1)$$
$$+ pE[(D_1 + D_2 + \ldots + D_{n-1}) - q]^+ + pE\left[D_n + \left[\ldots + \left[D_2 + \left[D_1 - q\right]^-\right]^-\right]^-\right]^+,$$

where the individual parameters are the following:

c_f – fixed cost. This cost always exists when the production of a series is started. [Ft / production]

c_v – variable cost. This cost type expresses the production cost of one product. [Ft / product]

p – penalty cost (or back order cost). If there is less raw material in the inventory than needed to satisfy the demands, this is the penalty cost of the unsatisfied orders. [Ft / product]

h – inventory and stock holding cost. [Ft / product]

D – this means the demand by the receiver for the product, which is an optional probability variable. [number / period]

$E[D]$ – expected value of the D stochastic variable.

q – product quantity in the inventory. The decision of the inventory control policy concerns the product quantity in the inventory after the product decision. This parameter includes the initial inventory as well. If nothing is produced, then this quantity is equal to the initial quantity, i.e. concerning the existing inventory.

I – initial inventory level. We assume that the supplier possesses I products in the inventory at the beginning of the demand of the delivery period.

n – number of periods

The new method is robust and adequately elegant (detailed in papers [6][7]), because the solution is independent from the type of the distributed function:

$$F_{123.n}(q^*) = \frac{p - c_v - hF_1(q^*) - hF_{12}(q^*) - hF_{123}(q^*) - \ldots - hF_{123.n-1}(q^*)}{h+p}, \quad (1.2)$$

where $F()$ represents the joint distribution function in compliance with the number of periods drawn together. q^*, which satisfies the equation, expresses how many finished products should be in the inventory at the time when customer demand appears with regard to n periods. Naturally, the critical inventory level, which was first mentioned by Herbert Scarf [1] for one-period production, can be applied in the case of joint production for n numbers of production cycles, as well. However, the present paper does not deal with this.

2. One Product, Multi-Period Model

When applying the global capacity constraint for the multi-period extended newsvendor model we take the characteristic features of the model and the period-based policy into consideration. Accordingly, two types of optimization methods can be distinguished: *service-level-based policy* and *cost-based policy*. Naturally, these policies are in contrast with each other. Only one of them can be considered in the stockpiling policy.

2.1. Cost-Based Policy

This approach models the type of supplier that is directly in connection with the market. In this policy, the main objective is to minimize the costs. It should be decided how many back-orders can be placed in the specified period of time. So the penalty cost is determined by the supplier itself [7]. Since in case of cost-based policy keeping the service level is not the main objective, the solution for the optimization of production costs can be *reducing the number of setups* or *taking the risk of penalty*.

The reduction of the number of jointly produced periods is not definitely the best solution and does not result in the minimization of costs. It is possible that paying a penalty cost is a cheaper way for the supplier.

Theorem 1: if the sum of the penalty cost appearing at producing the quantity according to the capacity constraint and the holding cost of quantity of the truncated period storing from the beginning of the time horizon, is lower than the cost of preparing a new setup, then taking the risk of the penalty is the proper policy.

We prove this theorem as follows. We denote the cost value of the back-orders by variable b. The following equation helps to decide what policy should be applied.

$$\text{if} \begin{cases} b + \sum_{i=1}^{a} h \cdot (C - q_a) \le c_f, \text{ then take the risk}, \\ b + \sum_{i=1}^{a} h \cdot (C - q_a) > c_f, \text{ then reduce the periods}. \end{cases}, \tag{2.1}$$

where h is a cumulative holding cost per product and $q_a < C$ is the quantity in compliance with the number of jointly produced periods. Variable a means the period number, q_a value is even smaller than capacity constraint C. Then

$$h \cdot a \cdot (C - q_a) = \sum_{i=1}^{a} h \cdot (C - q_a) \tag{2.2}$$

represents the holding cost, which appears as the difference between the quantity of capacity constraint and quantity of the reduced jointly produced periods. The formula means: if the value of $b + h \cdot a \cdot (C - q_a)$ is lower than a new setup cost (c_f), the cost will be minimized, if the supplier chooses to pay the penalty and produces the quantity of the capacity constraint. Otherwise reducing the number of the jointly produced periods is a good policy. Figure 1 shows the method of cost based policy.

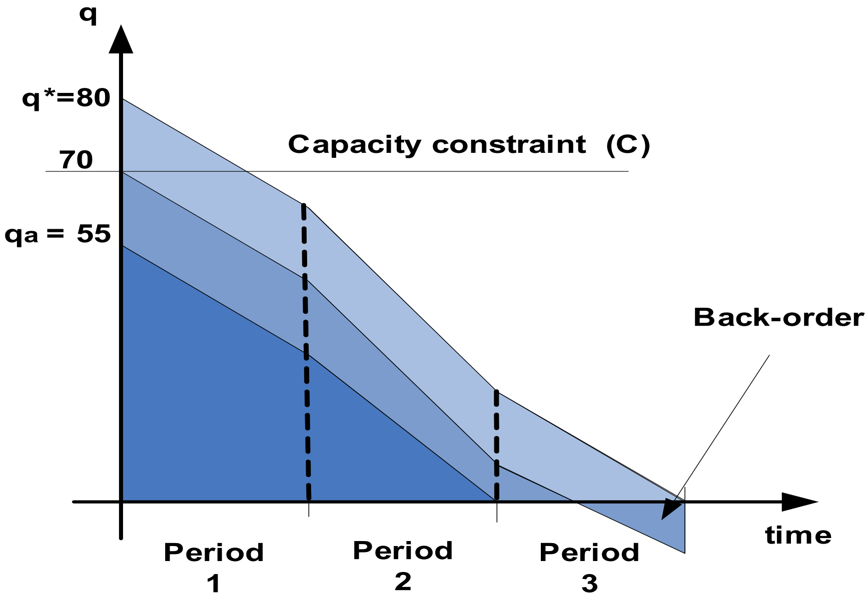

Figure 1. Applying capacity constraint in case of cost base policy

2.2. Service Level Based Policy

The main objective when choosing this policy is to ensure the predetermined *Service Level*. This level is determined in compliance with the objectives of the company. Applying capacity constraint means that the number of unsatisfied orders should be lower than the predetermined service level. Disregarding this important rule gets the relationship between the customer and the supplie at riskr.

The reduction of jointly produced periods is the solution for this problem. If the optimal quantity calculated with the extended newsvendor model exceeds the value of capacity constraint, then the predetermined service level can only be maintained if we reduce the number of jointly produced periods until the quantity satisfies the capacity condition according to the reduced period. This solution is justified by the unit cost variation curve which is further detailed in paper [7].

3. The Multi-Product, Multi-Period Model

Concerning this model, to solution capacity of constraint problems is most complicated. As a rule the ABC method is widely offered to solve the problem. On the basis of the Pareto diagram about the 'significance' distribution of the elements of a product set [4], several conclusions can be drawn. But the method does not give a proper answer to the questions arising while calculating the optimal stockpiling quantities.

Next we will present a new heuristic method to solve multi-product, multi-period and service-level-based capacity constraint optimization problems. We assume a global capacity constraint, which means that different products share one common production capacity and that the decisions of the inventory control policy are made for a long period of time.

We prefer in the solution presented the Service-Level-based policy. The objective is to determine the reduced number of jointly produced periods per product in a way that the sum of the total quantities should satisfy the capacity constraint condition.

The main idea behind this heuristic solution is the specific property of the unit cost of the products. Figure 2 shows the changes in the unit cost of a product against jointly produced periods.

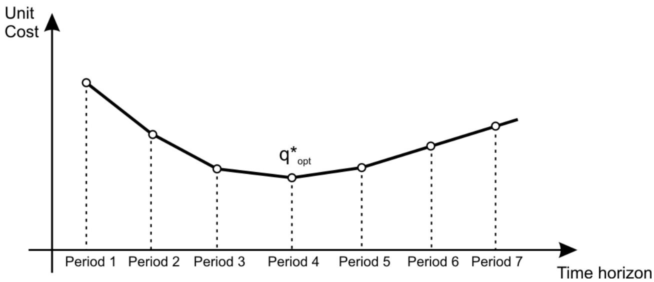

Figure 2. Unit cost changes in case of seven-period length time

Each product has a similar unit curve [7]. If the sum of the quantities of *n* number of products is larger than the value of capacity constraint then the solution should be changed. If

$$\sum_{i=1}^{n} u^i \cdot (q^{i*}_{opt_j} - I^i) \leq C \text{, the solution is optimal.} \qquad (3.1)$$

In the equation *i* (i=1,…,*n*) means the number of products, $q^{i*}_{opt_j}$ is the optimal quantity of the product *i*: opt_j means the number of jointly produced periods and u^i represents the capacity usage of the product. I^i denotes the initial inventory of

product *i*. We should determine the number of setups to satisfy the minimal cost conditions.
First we should start examining the unit cost curve. The following figure shows the changes in the unit cost for 4 periods, as a modification of Figure 2.

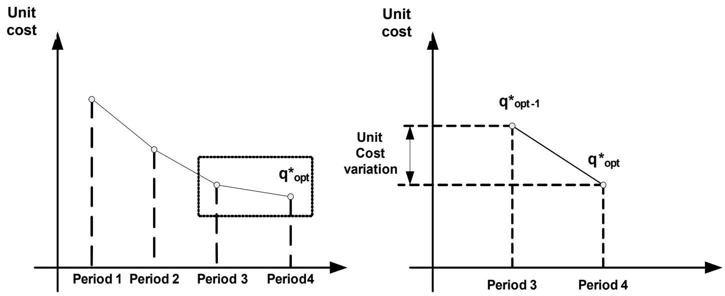

Figure 3. Increase in unit cost against jointly produced weeks

In Figure 3 it is easy to see, if the optimal number (4) of jointly produced periods is reduced to three periods, the value of the unit cost is bound to increase. This observation suggests the following:
Theorem 2: the capacity constraint can be regarded as the optimum solution when it is due to the reduction of jointly produced periods; there is a sum of minimum increases in the sum of the unit cost.

FKV_i denotes the sum of unit cost changes for product *i*. Then:

$$\sum_{i=1}^{n} FKV_i \longrightarrow \min \ . \qquad (3.2)$$

Theorem 2 helps to find the optimal solution, but a searching method is necessary, with which we can calculate the sum of unit cost changes fast in a multi-product, multi-period environment. In the following we present a new and suitable algorithm.

3.1 Algorithm and Other Parts of the Method

The basic idea behind our algorithm is the existence of the optimal solution per product without the capacity constraint condition. The objective of the method is to move the searching space along the minimal unit cost changes, because as we have mentioned before, the optimal solution means the minimal sum of per-unit variation costs. The algorithm can be divided into three main parts: (1) checking the capacity constraint condition (2) selecting optimum modifications possible and (3) choosing a combination to obtain a better solution.

While searching for the proper solution, these steps are continually repeated until the optimal supplier policy can be found in compliance with the capacity constraint conditions.

3.1.1 Capacity constraint condition test

The first step of the method is to determine the optimal number of jointly produced periods based on the introduced unit cost model [8]. This operation is performed only once during the running at the beginning. After that it should be investigated if there are any products the production of which can be 'shifted'. This can be achieved by comparing the quantities in the inventory and the optimal quantities according to the jointly produced periods. Regarding the first period:

$$c_v\left(q_j^{i*} - I_1^i\right) \leq 0\ ,\ j = 1,2,...,\ m\ ,i = 1,2,...,\ n \tag{3.3}$$

If a product can be found for which this equation is true during the calculation of the unit costs, then its production can be shifted forward along the time. After this, these products do not take part in the further steps.
The next step is the evaluation of the following capacity constraint condition.

$$\sum_{j=1}^{n} u^i q^i_{opt\ j - L_i} - I^i \leq C\ . \tag{3.4}$$

If the condition is satisfied, then we have the optimum solution. The equation has a new element L_i, which modifies the optimal number of jointly produced periods. L_i represents the solution vector and means the reduced jointly produced periods of the products in the iteration steps of the algorithm. At the beginning of the iteration, this is a zero vector. If the equation is not fulfilled the next step follows.

3.1.2 Selecting the optimum modifications possible

If the solution in the first step or in a previous iteration does not satisfy the capacity constraint, then a modification of the solution is required. In the second step of the algorithm we will choose the products which can be suitable in determining the optimal solution. Choosing the optimum modification possible always means the product which has a minimum unit cost variation. To determine this product the following steps must be taken: the formerly calculated optimal number of jointly produced periods is reduced virtually by one period considering the current modifications (L_i). For product *i* it means: $opt_j - (L_i + 1)$.

Before and after the reduction, we can calculate both the unit cost and changes in the unit cost variation.

We will choose only one product which has the minimum unit cost variation in this iteration step. If we have chosen product *i*, then we will increase the *i.th* element of the solution vector: $L_i = L_i + 1$. The following figure shows this reduction method.

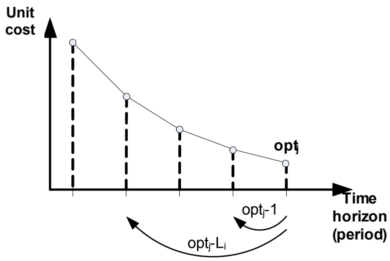

Figure 4. Reduction of the jointly produced periods against the solution vector

The product with the maximum unit cost variation will also be chosen if the combination was performed in the previous iteration. This choice constitutes the basis for the last step of the algorithm, which forbids infinite iteration loops (detailed in step 3).

3.1.3 Selecting a combination for a better solution

Selecting the minimal setup cost is not enough to find the best solution. There can be cases, when the sum of setup cost variation of two products can be substituted for per-setup cost variation of another product in order to achieve a better solution. Figure 5 presents changes in the setup cost of three products and the possibility of substitution.

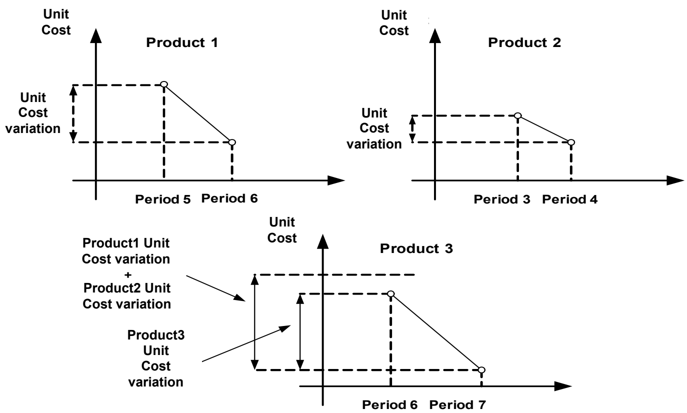

Figure 5. Comparison and substitution of setup cost increases

Explanation of Figure 5: because the originally computed solution does not meet the capacity constraint condition, the number of jointly produced periods is reduced by this algorithm. According to the first step of the algorithm, the product with the minimal variation value of unit cost will be chosen. We will choose the first product. Let us suppose that the solution after the reduction of jointly produced periods from six to five still does not meet the capacity constraint. In case of one product, the substitution phase cannot be explained, so the algorithm runs on. In the second step we will choose another product, with the minimal variation value of unit cost, which will be the second product now.

It is not sure that the reduction of jointly produced periods of the two chosen products is the optimal solution. Therefore we should examine if the sum of the increase of unit cost variations, resulting from the reduction of the jointly produced periods of the two chosen products, can be substituted for a smaller variation of unit cost. Figure 5 shows that the value of unit cost of product three is lower than the variations sum of product one and two.

This means that the variations sums regarding products one and two can be substituted by a reduction of jointly produced periods at product three. Let us examine what will happen if there are four products. In the next figure, the unit cost variations of product three and four can be seen. The situation shows one period decrease of its jointly produced periods.

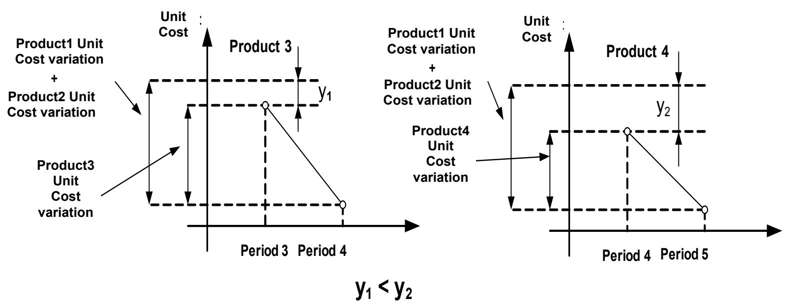

Figure 6. Substitution phase in case of more than three products

In case of more than three products, the question arises: which product should be chosen for substitution. In Figure 6 we can see that both variations of unit costs for product three and four are smaller than the variations sum for the first two products.

In this case the product should be chosen, where the variation is the farthest from the variations sum. The main reason for this is following: if the result of the substitution does not meet the capacity constraint, the algorithm in the next step chooses another product, with the minimal unit cost variation. In this example, the first product satisfies the condition. Let us suppose that this is the optimum

solution. If we choose product four, then the sum of unit cost variations is certainly smaller if we choose product three for substitution.
During the substitution process we use the product with maximum unit cost variation value found in the second step. This value and product constitute the basis of reference in the investigation of the unit cost variations. We will examine as reference the possibility of merging according to this value because it cannot be the chosen product.
If the substitution is carried out successfully it is necessary to prepare the next iteration. The first step is to modify the solution vector. The value in the vector belonging to the selected product should be set to zero. This ensures that the algorithm can move the search space along the changes in the minimal unit cost.
After this process the next iteration comes until the solution meets the capacity constraint condition.
Calculations in practice show clearly that to find the optimal solution we do not need a lot of iteration steps. In case of a product, the optimal number of jointly produced periods is about 7-8 periods. The analytic solution for the extended newsvendor model ensures high-performance calculation in an optional multi-product, multi-period environment for a long period of time.

4. Conclusion

In this paper we extended the previously elaborated and modified newsvendor model [6][7] with the condition of global capacity constraint. Based on the periodic feature of the model, two problem groups were distinguished and presented. For the most complex, multi-period, multi-product case a new heuristic method was elaborated. This model enables the determination of a cost-optimal stockpiling policy applying capacity constraint in case of an arbitrary product number and an arbitrary length of production time. Because of the specific approach of the new model, it can be used effectively in practice compared with other models.

Acknowledgements

The research and development summarized in this paper have been carried out by the Production Information Engineering and Research Team (PIERT) established at the Department of Information Engineering. The research is supported by the Hungarian Academy of Sciences and the Hungarian Government with the NKFP VITAL Grant. The financial support of the research by the aforementioned sources is gratefully acknowledged. Special thanks to Ferenc Erdélyi for his valuable comments and review work.

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